On the Casimir energy for scalar fields with bulk inhomogeneities

I.V. Fialkovsky[a*], V.N. Markov[b†], Yu.M. Pis'mak[a‡]

[a)] St Petersburg State University, St Petersburg, Russia

[b)] St Petersburg Nuclear Physics Institute, Gatchina, St Petersburg, Russia

July 4, 2008

Abstract

We study the field theoretical model of a real scalar field in presence of spacial inhomogeneity in form of a finite width mirror (material layer). The interaction of the scalar field with the defect is described with position-dependent mass term. We calculate the propagator of the theory, the Casimir energy and the pressure on the boundaries of the layer. We discuss the renormalization procedure for the model in dimensional regularization.

1 Introduction

Quantum Field Theory (QFT) was developed in the middle of the last century as a theory of interaction of elementary particles in otherwise empty, homogenous infinite space-time [1]. On the other hand, from the very beginning it was clear that presence of boundaries, non-zero curvature or nontrivial topology of the space-time manyfold should influence the spectrum and dynamics of the excited states of the model as well as the properties of the ground state (vacuum).

The first quantitative description of such changes in the vacuum properties was made by H. Casimir in 1948. He predicted [2] macroscopical attractive force between two uncharged conducting plates placed in vacuum. The force appears due to the influence of the boundary conditions on the electromagnetic quantum vacuum fluctuations. Nowadays the Casimir effect is verified by experiments with the precision of 0.5% (see [19] for a review).

The properties of the vacuum fluctuations in curved spaces, investigation of scalar field models with various boundary conditions and their application to the description of

*e-mail: ignat.fialk@paloma.spbu.ru
†e-mail: markov@thd.pnpi.spb.ru
‡e-mail: pismak@JP7821.spb.edu

real electromagnetic effects were actively studied through the last decades, see discussion and references in [19], [20].

However, it was well understood that boundary conditions must be considered just as an approximate description of complex interaction of quantum fields with the matter. A generalization of the boundary conditions method has been proposed by Symanzik [3]. In the framework of path integral formalism he showed that presence of material boundaries (two dimensional defects) in the system can be modeled with a surface term added to the action functional. Such singular potentials with δ-function profile concentrated on the defect surface reproduce some simple boundary conditions (namely Dirichlet and Neumann ones) in the strong coupling limit. The additional action of the defect should not violate basic principles of the bulk model such as gauge invariance (if applicable), locality and renormalizability.

The QFT systems with δ-potentials are mostly investigated for scalar fields. In [4]–[6] the Symanzik approach was for the first time used to describe similar problems in complete quantum electrodynamics (QED), and all δ-potentials consistent with QED basic principles were constructed.

It seems quite natural to try applying the same method for description of interaction of quantum fields with bulk macroscopic inhomogeneities (slabs, finite width mirrors, etc) and to study Casimir effects in system of such a kind. There were different attempts to quantize electrodynamics in presence of dielectric media (i.e. volume inhomogeneities of special kind) see, for instance, [8], [9], none of them was truly successful. The Symanzik's method was used to model the interaction of quantum fields with bulk defects in a number of papers (e.g. [10],[12]-[15], and others). However most of them were devoted to study of a limiting procedure of transition from a bulk potential of the defect to the surface δ-potential as in [12]. On the other hand, results for the Casimir energy of a single planar layer of finite width ℓ are contradictory. Thus, the formulae presented recently in [16] does not coincide with previous calculations made in [10]. Moreover, the only attempt to calculate the propagator in such system was undertaken in [17] where hardly any explicit formulae were after all presented.

Thus, one can see that the specificity of finite volume effects generated by inhomogeneities in QFT has not been yet adequately explored. Our work is dedicated to clarify the problem, and to solve existing controversy within an accurate and unambiguous approach. We consider a model of massive scalar field interacting with volume defect (finite width slab), calculate the modified propagator of the field, the Casimir energy of the slab and discuss its physical meaning.

2 Statement of problem

Let us consider a model of a real scalar field interacting with a volume defect. In the simplest case such defect could be considered as homogenous and isotropic infinite plane layer of the thickness ℓ, placed in the x_1x_2 plane. Generalizing the Simanzik approach, we describe the interaction of quantum fields with matter by introducing into the action

of the model an additional mass term which is non-zero only inside the defect

$$S = \frac{1}{2}\int d^4x\left(\phi(x)(-\partial_x^2 + m^2)\phi(x) + \lambda\theta(\ell, x_3)\phi^2(x)\right) \tag{1}$$

where $\partial_x^2 = \partial^2/\partial x_0^2 + \ldots + \partial^2/\partial x_3^2$[1]). The distribution function $\theta(\ell, x_3)$ is equal to $1/\ell$ when $|x_3| < \ell/2$, and is zero otherwise, in terms of the Heaviside step-function we can write it as $\theta(\ell, x_3) \equiv [\theta(x_3 + \ell/2) - \theta(x_3 - \ell/2)]/\ell$. Such kind of potential is also called patchwise (or piecewise) constant one. In the framework of QFT it was considered for the first time in [10], and later in [13]-[16].

To describe all physical properties of the systems it is sufficient to calculate the generating functional for Green's functions

$$G[J] = N\int D\phi\,\exp\{-S[\phi] + J\phi\},\ N^{-1} = \int D\phi\,\exp\{-S_0[\phi] + J\phi\} \tag{2}$$

where J is an external source, $S_0(\phi) = S(\phi)|_{\lambda=0}$, and normalization for the generating functional we have chosen in such a way that $G[0]|_{\lambda=0} = 1$.

Introducing in (2) auxiliary fields ψ defined in the volume of the defect only, we can present the defect contribution to $G[J]$ as

$$\exp\left\{-\frac{\lambda}{2\ell}\int d\vec{x}\int_{-\ell/2}^{\ell/2} dx_3\phi^2(x)\right\} = C\int D\psi\,\exp\left\{\int d\vec{x}\int_{-\ell/2}^{\ell/2} dx_3\left(-\frac{\psi^2}{2} + i\sqrt{\kappa}\psi\phi\right)\right\} \tag{3}$$

where C is an appropriate normalization constant, and $\kappa = \lambda/\ell$.

With help of projector onto the volume of defect $\mathcal{O} = \theta(x_3 + \ell/2) - \theta(x_3 - \ell/2)$ acting as

$$\psi\mathcal{O}\phi \equiv \int d\vec{x}\int_{-\ell/2}^{\ell/2} dx_3\psi\phi,$$

we can perform functional integration over ϕ, and consequently over ψ. As a result we get

$$G[J] = [\mathrm{Det}Q]^{-1/2}e^{\frac{1}{2}J\hat{S}J},\quad \hat{S} = D - \kappa(D\mathcal{O})Q^{-1}(\mathcal{O}D), \tag{4}$$

$$Q = \mathbf{1} + \kappa(\mathcal{O}D\mathcal{O}). \tag{5}$$

Here the unity operator $\mathbf{1}$, as well as the whole Q, is defined in the volume of the defect only $(-\ell/2, \ell/2) \times \mathbb{R}^3$, and $D = (-\partial^2 + m^2)^{-1}$ is the standard propagator of free scalar field. We shall note here that the outlook of (4) completely coincides with expression for generating functional $G[J]$ in the case of delta-potential term instead of patchwise constant one. It is also evident that a straightforward generalization is possible for non-constant $\kappa(= \lambda/\ell)$ with λ depending on x_3.

In this paper we calculate explicitly both the modified propagator of the system and its Casimir energy, and reveal their dependence on the parameter λ describing the material properties of the homogeneous defect layer and its thickness ℓ.

[1]We operate in Euclidian version of the theory which appears to be more convenient for calculations.

3 Calculation of the propagator

To calculate the propagator $\hat{S}$ defined according to (4) let us first derive explicit formula for the operator $W \equiv Q^{-1}$.

For this purpose we first introduce the Fourier transformation of the coordinates parallel to the defect (i.e. x_0, x_1, x_2). Then for the propagator D of the system without a defect one can write

$$D(x) = \int \frac{d^3\vec{p}}{(2\pi)^3} e^{i\vec{p}\vec{x}} \int \frac{dp_3}{(2\pi)} \frac{e^{ip_3x_3}}{p_3^2 + \vec{p}^2 + m^2},$$

integrating over p_3 with help of the residue theorem we get

$$D(x) = \int \frac{d^3\vec{p}}{(2\pi)^3} e^{i\vec{p}\vec{x}} \mathcal{D}_{E^2}(x_3), \qquad \mathcal{D}_V(x) \equiv \frac{e^{-\sqrt{V}|x|}}{2\sqrt{V}} \tag{6}$$

with $E = \sqrt{p^2 + m^2}$. Then we are able to write the defining (operator) equation for W as

$$W + \kappa \mathcal{D}_{E^2} W = 1. \tag{7}$$

By construction the mixed $\vec{p}$-x_3 representation of the free scalar propagator $\mathcal{D}_{E^2}(x,y) \equiv \mathcal{D}_{E^2}(x-y)$ is the Green's function of the following ordinary differential operator

$$K_V(x,y) = \left(-\frac{\partial^2}{\partial x^2} + V \right) \delta(x-y) \tag{8}$$

for $V = E^2$. Multiplying both sides of (7) with K_{E^2} and using obvious relation $K_V - K_{V'} = V - V'$ we get

$$K_\rho U = -\kappa \tag{9}$$

where $\rho \equiv \kappa + E^2$ and $U \equiv W - 1$.

The general solution to this (inhomogeneous) operator equation can be written as a sum of its partial solution and the general solution of its homogeneous version. Then with help of $\mathcal{D}_\rho$ one writes for U

$$U(x,y) = -\kappa \mathcal{D}_\rho(x,y) + \alpha(y) e^{x\sqrt{\rho}} + \beta(y) e^{-x\sqrt{\rho}}.$$

Here α and β — arbitrary functions on y. Imposing the symmetry condition $U(x,y) = U(y,x)$ we derive that

$$U(x,y) = -\kappa \mathcal{D}_\rho(x,y) + a e^{(x+y)\sqrt{\rho}} + b \left(e^{(x-y)\sqrt{\rho}} + e^{(y-x)\sqrt{\rho}} \right) + c e^{-(x+y)\sqrt{\rho}}$$

where a, b and c are some constants now. Introducing $W = 1 + U$ into (7) one gets

$$U + \kappa \mathcal{D}_{E^2}(1 + U) = 0. \tag{10}$$

Requiring that this equation is an identity for all x and y (we remind that $U \equiv U(x,y)$), we find for a, b and c

$$a = c = -\frac{\xi\kappa^2 e^{\ell\sqrt{\rho}}}{2\sqrt{\rho}}, \qquad b = -\frac{\xi\kappa(E-\sqrt{\rho})^2}{2\sqrt{\rho}}, \tag{11}$$

$$\xi = \frac{1}{e^{2\ell\sqrt{\rho}}(E+\sqrt{\rho})^2 - (E-\sqrt{\rho})^2}.$$

With help of these expressions we can finally derive the explicit formula for the modified propagator of the system. From the definitions of $\hat{S}$ and W, and using (10) we can write that

$$\hat{S} = (1+U)\mathcal{D}_{E^2}. \tag{12}$$

We divide the general expression of $\hat{S} \equiv \hat{S}(\vec{p}, x_2, y_3)$ into four parts according to the position of x_3, y_3 relative to the defect

$$\hat{S}(\vec{p}, x_3, y_3) = \begin{cases} S_{--}, & x_3 < -\ell/2,\ y_3 < -\ell/2 \\ S_{-\circ}, & x_3 < -\ell/2,\ y_3 \in (-\ell/2, \ell/2) \\ S_{-+}, & x_3 < -\ell/2,\ y_3 > \ell/2 \\ S_{\circ\circ}, & x_3 \in (-\ell/2, \ell/2),\ y_3 \in (-\ell/2, \ell/2) \end{cases} \tag{13}$$

Other cases could be easily derived using the symmetry properties of the propagator.

Performing necessary integration according to (12) we get

$$\begin{aligned} S_{--} &= \mathcal{D}_{E^2}(x_3 - y_3) + \frac{\xi\kappa e^{E\ell}(1-e^{2\ell\sqrt{\rho}})}{2E} e^{E(x_3+y_3)} \\ S_{-\circ} &= \xi e^{E(x_3+\ell/2)} e^{\ell\sqrt{\rho}} \left((\sqrt{\rho}-E)e^{\sqrt{\rho}(y_3-\ell/2)} + (\sqrt{\rho}+E)e^{\sqrt{\rho}(\ell/2-y_3)}\right) \\ S_{-+} &= 2\xi\sqrt{\rho} e^{(\sqrt{\rho}+E)\ell + E(x_3-y_3)} \\ S_{\circ\circ} &= \frac{\xi e^{\ell\sqrt{\rho}}}{2\sqrt{\rho}} \left(2\kappa\cosh[(x_3+y_3)\sqrt{\rho}] + e^{\sqrt{\rho}(|x_3-y_3|-\ell)}(E-\sqrt{\rho})^2 + e^{\sqrt{\rho}(\ell-|x_3-y_3|)}(E+\sqrt{\rho})^2\right) \end{aligned} \tag{14}$$

with ξ defined in (11).

To the best of our knowledge the only attempt to calculate the propagator for such system was presented in [17] where its final expression was given in terms of "coefficients of scattering wave functions" of one-dimensional time-dependent Schrodinger equation. However, for the explicit formulae for those coefficients the author refers yet to another paper [18] (actually, there is also a misprint in the reference number), where the problem of electrons scattering in a powerful laser field is considered and corresponding coefficients are presented in the from of infinite series of Bessel functions. The result presented in (14) is in much simpler closed from, and it raises doubts of correctness of calculations presented in [17], [18].

4 The Casimir Energy

It is well know that the Casimir energy density per unit area of the defect S can be presented with the relation

$$\mathcal{E} = -\frac{1}{TS}\ln G[0] = \frac{1}{2TS}\,\mathrm{Tr}\ln[Q(x,y)]. \tag{15}$$

in the second equation we used (4). For explicit calculations we first make the Fourier transformation as in (6). Then

$$\mathcal{E} = \mu^{4-d}\int \frac{d^{d-1}\vec{p}}{2(2\pi)^{d-1}}\mathrm{Tr}\ln[Q(\vec{p};x_3,y_3)], \tag{16}$$

where we also introduced dimensional regularization to handle UV-divergencies and an auxiliary normalization mass parameter μ.

Using the definitions of U and Q we can express κ-derivative of the integrand of (16) in the following form

$$\partial_\kappa \ln Q = \mathcal{D}_{E^2}W = -\frac{U}{\kappa}.$$

Then for the energy density we get

$$\mathcal{E} = -\mu^{4-d}\int_0^\kappa \frac{d\kappa}{\kappa}\int \frac{d^{d-1}\vec{p}}{2(2\pi)^{d-1}}\mathrm{Tr}\,U. \tag{17}$$

We have chosen the lower limit of integration over κ to satisfy the energy normalization condition $\mathcal{E}|_{\kappa=0}=0$. As we show below the integral is convergent at $\kappa = 0$.

The trace of the integral operator U is straightforward

$$\mathrm{Tr}\,U \equiv \int_{-\ell/2}^{\ell/2} dx\,U(x,x) = 2b\ell + \frac{4a\sinh(\ell\sqrt{\rho}) - \ell\kappa}{2\sqrt{\rho}} \tag{18}$$

where we already used that $a = c$. Using a and b given in (11), one easily notes that $\mathrm{Tr}U \sim -\ell\kappa/(2E)$ when $\kappa \to 0$, thus supporting the above statement.

Next, putting (18) into (17) we can compare our result with previous calculations performed in [10], and also recently rederived in [11]. Instead of explicit κ-integration in (17), we can differentiate the above mentioned result by Bordag with respect to κ to see immediately that it coincides explicitly with integrand of (17). Thus, we write for the energy

$$\mathcal{E} = \mu^{4-d}\int \frac{d^{d-1}\vec{p}}{2(2\pi)^{d-1}}\ln\left[\frac{e^{-\ell E}}{4E\sqrt{\rho}}\left(e^{\ell\sqrt{\rho}}(E+\sqrt{\rho})^2 - e^{-\ell\sqrt{\rho}}(E-\sqrt{\rho})^2\right)\right] \tag{19}$$

To extract the UV divergencies in $d=4$, let's consider those contributions in $\mathcal{E}$ (19) that do not converge while integrated over p. We have

$$\ln\left[\frac{e^{-\ell E}}{4E\sqrt{\rho}}\left(e^{\ell\sqrt{\rho}}(E+\sqrt{\rho})^2 - e^{-\ell\sqrt{\rho}}(E-\sqrt{\rho})^2\right)\right] = \frac{\lambda}{2E} - \frac{\lambda^2}{8\ell E^3} + O\left(\frac{1}{E^4}\right),$$

Hence, within dimensional regularization the energy can be represented as follows

$$\mathcal{E} = \mathcal{E}_{fin} + \mathcal{E}_{div},$$

where

$$\mathcal{E}_{fin} = \frac{1}{2\pi^2}\int_0^\infty \Xi(p)p^2 dp,$$

$$\Xi(p) \equiv \ln\left[\frac{e^{-2\ell E}}{4E\sqrt{\rho}}\left(e^{2\ell\sqrt{\rho}}(E+\sqrt{\rho})^2 - e^{-2\ell\sqrt{\rho}}(E-\sqrt{\rho})^2\right)\right] - \frac{\lambda}{4E}\left(1-\frac{\lambda}{4\ell E^2}\right), \qquad (20)$$

$$\mathcal{E}_{div} = \frac{\lambda\mu^{4-d}}{2(2\pi)^{d-1}}\int \frac{d^{d-1}p}{4E}\left(1-\frac{\lambda}{4\ell E^2}\right).$$

The first item $\mathcal{E}_{fin}$ is finite and we removed regularization, while $\mathcal{E}_{div}$ is divergent but trivially depends on the parameters of the theory and auxiliary parameter μ. We add now to the action of the model a field-independent counter-term δS of the form $\delta S = f + g\ell^{-1}$, with bare parameters f and g (of mass dimensions two and one correspondingly). It allow us to choose these parameters in such way that the renormalized Casimir energy $\mathcal{E}_r$ defined by the full action $S + \delta S$ and considered as the function of renormalized parameters appears to be finite both in regularized theory, and also after the removing of regularization.

Thus, for the renormalized Casimir energy we obtain the following result

$$\mathcal{E}_r = \mathcal{E}_{fin} + f_r + \frac{g_r}{\ell} \qquad (21)$$

where finite parameters f_r, g_r must be determined with appropriate experiments.

The Casimir pressure on the slab is then

$$p = -\frac{\partial\mathcal{E}_r}{\partial\ell} = -\frac{\partial\mathcal{E}_{fin}}{\partial\ell} + \frac{g_r}{\ell^2}.$$

Taking into account the definition of distribution function $\theta(\ell, x_3)$ one can say that the derivative is taken here on condition that the amount of matter (effectively described by the defect) in the slab is fixed: $\int dx_3\theta(\ell, x_3) = 1$. Alternatively, one can consider the density of the matter to be fixed and calculate the pressure under this condition. Then the distribution function has a different normalization condition $\int dx_3\theta(\ell, x_3) = \ell$, which is equivalent to the mere change of variables $\lambda \to \ell\tilde{\lambda}$ in the formula (20).

5 Conclusion

We constructed QFT model of the scalar field interacting with the bulk defect concentrated within a slab of finite width ℓ. The propagator and the vacuum determinant (Casimir energy) were calculated. The later one coincides with results obtained in [10], [11] within a different approach, while the explicit formula for the propagator is given

for the first time. The Casimir energy is UV divergent and for its regularization we applied dimensional regularization. It allowed us to extract the finite part and to construct the counter-terms. The renormalization procedure requires generally two normalization conditions to fix the values of the counter-terms with the appropriate experiments. It is shown that the Casimir pressure in the system can be calculated in two different ways: for fixed density of matter and for fixed amount of matter of the slab.

Similar problems were considered recently in [16] in the framework of massless scalar field interacting with a slab (mirror) of general profile. However, the massless limit of our result for the Casimir energy of a single slab (19) differs from one obtained in [16], Eq. (68) for the case of 'piecewise constant' profile (equivalent to our case). As a validity check we appeal to the general perturbation theory. Decomposing the generating functional $G(J)$ (2) in a perturbation series in λ, one finds that for the massive theory $G(0)$ is analytical at $\lambda = 0$ with irrelevant (geometry independent) linear term. However, it is evident that naive perturbation expansion fails for the limit $m \to 0$, alerting us of non-analyticity of the vacuum energy at $\lambda = 0$. Expanding Eq. (68) of [16] in a power series in λ one can easily see that it is perfectly analytical with non vanishing linear term, and thus does not comply with this general argument. At the same time both the massive and massless limits of our result (17), which is equivalent to (19) derived independently by two other groups, does posses the required (non-) analyticity properties.

In our work we considered a model of interaction of quantum scalar field with material slab assuming $\lambda > 0$. One must note that with a simple redefinition of the parameters of the system under consideration (i.e. $\lambda = -2m^2\ell$) one can calculate the Casimir energy of two semi-infinite slabs separated by a vacuum gap and interacting through a massless scalar field. Similar problem in the framework of quantum statistical physics was first solved by Lifshitz, [7]. Comparison with Lifshitz formula, and further generalization of the method proposed in this paper to the case of QED is the scope of our future work.

Acknowledgement

Authors are grateful to Prof. Vassilevich for attracting our attention to some literature on the issue, and to Prof. Mazzitelli for his interest in our research.

V.N. Markov and Yu.M. Pismak are also grateful to Russian Foundation of Basic Research for financial support (RFRB grant 07–01–00692).

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